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# A MATHEMATICAL ANALYSIS OF THE DIVISION OF THE TRIBES AND THE ROLE OF THE LEVITES ON GRIZIM AND AVAL IN DEUTERONOMY 27

## INTRODUCTION

This article provides a mathematical analysis of the division of the tribes and the role of the Levites as described in Deuteronomy 27:11-14. These verses state:

11. And Moses commanded the people on the same day, saying.

12. These shall stand upon Mount Grizim to bless the people, when they pass over the Jordan: Shimon, Levi, Yehuda, Issachar, Yosef and Binyamin.

13. And these shall stand upon Mount Aval for the curse; Reuven, Gad, Asher, Zevulun, Dan and Naphtali.

14. And the Levites shall call out, and say unto all the people of Israel, with an exalted voice.

I.

The Bible in verses 12 and 13 divides the nation of Israel into two groups. One group consists of six tribes (Shimon, Levi, Judah, Issachar, Yosef, and Benjamin) standing on Mount Grizim. The other group of six tribes (Reuven, Gad, Asher, Zevulun, Dan, and Naphtali) stand on Mount Aval. Levi is thus apparently counted among the 12 tribes on the mountain, with Yosef counted as only one tribe (instead of separately counting each of Yosef's component members, Ephraim and Menashe).

There is, however, reason to advance an understanding of the text at variance with the literal reading of verses 12 and 13, which places all Levites on Grizim; the Bible has a special role for the Levites in the ceremony. In verse 14, the Levites are to "call out" to the people the curses and blessings. It would be anomalous if the same people "calling out" stood among one of the two groups being called to. It therefore seems logical that at least some of the Levites (i.e., those charged with "calling out") were positioned separately, between the mountains. In fact, the recounting of the actual blessing and cursing in the Book of Joshua even more strongly indicates that some Levites were in the valley. The Book of Joshua (8:33) states:

> All of Israel, their elders and their judges stood on this or that side of the ark *in front of the Priests the Levites that bore the ark* of the covenant of God, as well as the proselyted and born-Jew; half of them in front of mount Grizim and half in front of mount Aval as Moses the servant of God had commanded at the first to bless the people of Israel.

This verse, and the ambiguities found in Deuteronomy 27:11-14, motivate the Talmud to comment on the role of the Levites. The Babylonian Talmud (*Sotah* 32a & 37a) advances three different interpretations for the role played by the Levites:

(1) All the people—Levites and Israelites—were in the valley;
(2) The elders of the Priests and Levites were in the valley and the rest of the Levites were on the mountain; or
(3) Those Levites who were involved in working in the Tabernacle were in the valley and the rest were on the mountain.

Rashi, commenting on this third possibility, states that this refers to those Levites who were between the ages of 30 and 50 and thus worked in the Tabernacle. Maharsha disagrees and states that this third possibility is a reference to the family of Kehat whose duties encompassed moving the ark.

The Jerusalem Talmud (*Sotah* 7:4) adds one other possibility. It suggests that:

(4) The phrase "Levites" found in Deuteronomy 27:14 refers not to the Levites but to the priests. Only the priests were in the valley. All the Levites were on the mountain.

Explanation (1) involves resolving the textual difficulties by favoring one text over another, and placing *all* the Levites in the valley. Explanations (2), (3) and (4), on the other hand, resolve the difficulty by dividing the Levites into two different groups and thus allowing the literal understanding of each text to be satisfied.[1]

## II.

Another puzzling aspect of the positioning of the tribes is the particular manner in which the tribes were divided into two groups. The allocation chosen for this ceremony bears little or no relationship to the biblical schemes that are typically used for organizing the tribes, such as order of birth, matriarchal lineage, or the division into marching camps discussed in Numbers 2. In fact, there are 462 different ways to divide 12 tribes into two groups of six.[2] Why was the particular allocation described in these verses chosen?

This issue is addressed to some degree in the Babylonian Talmud (Sotah 36a-b) and in greater detail (and according to Tosafot, clarity) in the Jerusalem Talmud (*Sotah* 7:4). Five possibilities are suggested:

(1) The tribes were divided according to the pattern of their marching (with some changes);

(2) The tribes were divided roughly according to their listing in Exodus 1:2 (again with some changes);

(3) The tribes were divided according to the manner that they were listed on the High Priest's breast-plate (*efod*) (with some changes);

(4) The tribes were divided so as to have the same sum total number of letters in each of the six tribes names. (This does not work, the Talmud acknowledges, unless one uses non-standard spellings for the tribes' names); or

(5) The tribes were divided in some manner relating to their maternal linage (albeit with some changes).

As the Talmud and its commentaries recognize, none of these explanations are without question or work completely. In fact, a number of post-talmudic commentaries on the Bible adopt explanations at variance with those advanced in the Talmud for precisely this reason.[3]

## III.

In essence, this biblical text poses two pressing problems:

1. Where did the Levites stand during the ceremony?
2. Why were the tribes divided according to the particular allocation given in verses 12 and 13?

We propose a solution that resolves both of these difficulties with a single theory.

We suggest that the tribes and the Levites were assigned to the two mountains and the valley in such a way as to come as close as possible to a *numerically even* split while still allowing for a designated group of Levites to have a unique role calling out the blessings and curses in the valley. This mathematically optimal division occurs when the Levites are divided according to the Talmudic suggestion that "those Levites who were involved in working in the Tabernacle were in the valley and the rest were on the mountain." *What is most surprising is that this division is mathematically optimal whether the explanation of Rashi or of Maharsha is adopted as to which Levites are members of this group.* Both of the allocations advocated by Rashi and Maharsha, in fact, do optimally minimize the difference in population between the two groups, since both of these explanations—and only these two explanations—provide that between 8,770 and 8,970 Levites stood between the mountains to "call out" the blessing and the curse, while the remainder of the Levites stood on Mount Grizim.[4]

According to Rashi's explanation that those who worked in the Tabernacle refers to those aged 30 to 50, in an earlier census this group comprised 38.48% of the total number of Levites counted.[5] It is reasonable to assume, in the absence of any evidence to the contrary, that an approximately similar percentage of Levites were aged 30 to 50 at the end of the 40 years in the desert. Thus, there would be 8,850[6] Levites between the ages of 30 and 50. The group of Levites on the mountain would be all the remaining Levites, and their population would be 14,150.

Population figures for the two mountain groups, based on the census of Numbers 27, would be as follows:

| Tribes on Mt. Grizim | | Tribes on Mt. Aval | |
|---|---|---|---|
| Shimon | 22,200 | Reuven | 43,730 |
| Levi | 14,150 | Gad | 40,500 |
| Yehuda | 76,500 | Asher | 53,400 |
| Issachar | 64,300 | Zevulun | 60,500 |
| Yosef | 85,200 | Dan | 64,400 |
| Binyamin | 45,600 | Naphtali | 45,400 |
| Total 307,950 | | Total 307,930 | |

Total population = 615,880

Under this division, 50.001623% of the nation are on Grizim, and 49.998376% are on Aval.

According to Maharsha's explanation that those who worked in the Tabernacle refers to the Kehat family, in an earlier census this group is numbered 8,600 and comprised 38.57% of the total number of Levites counted.[7] Again, assuming the percentage of Kehat members remains constant, there would be 8,871 members of the Kehat family at the time

of the calling out. The group of Levites on the mountain would be all the remaining Levites, and their population would be 14,129.

Population figures for the two mountain groups, based on the census of Numbers 27, would be as follows:

| Tribes on Mt. Grizim | | Tribes on Mt. Aval | |
|---|---|---|---|
| Shimon | 22,200 | Reuven | 43,730 |
| Levi | 14,129 | Gad | 40,500 |
| Yehuda | 76,500 | Asher | 53,400 |
| Issachar | 64,300 | Zevulun | 60,500 |
| Yosef | 85,200 | Dan | 64,400 |
| Binyamin | 45,600 | Naphtali | 45,400 |
| Total | 307,929 | Total | 307,930 |

Total population = 615,859

Under this division, the tribes are nearly perfectly divided. In fact the percentage difference of this division (307,929 on mountain and 307,930 on the other) is insignificant, and it could be argued that the Bible's methodology of expressing these numbers in round figures would lead one to conclude that this division is perfect.

By means of a computer program, all 462 possible allocations of the tribes into two groups of six were generated. A complete list of all 462 combinations, sorted in order of the resulting numerical differences between populations on the two mountains, is provided in the appendix. The biblically prescribed allocation of the tribes (Shimon, Levi, Yehuda, Issachar, Yosef and Binyamin on one side, and Reuven, Gad, Dan, Naphtali, Asher, and Zevulun on the other) is the optimal division—of the 462 different possibilities, this one divides the tribes into the two most equal camps of six tribes each. No better allocation is possible, whether one adopts the understanding of Rashi or Maharsha as to which division of the Levites is correct. No other textually supportable division of the Levites does.[8]

Moreover, by examination of the two next best allocations, which respectively result in either 220 extra or 180 fewer people in the group of tribes which includes the Levites, it follows that the biblical allocation remains optimal (in terms of population equality) if and only if the number of Levites in that group is not decreased by more than 120 (in which case the division SLYJDN found in the appendix becomes optimal) or increased by more than 80 (in which case the division SJGRZD or SLYJBD found in the appendix becomes optimal). Thus, we may conclude that as long as between 14,030 and 14,230 Levites stood on Grizim (with the remaining 8,770 to 8,970 "calling out" ministerially below), the biblical allocation remains the most optimal of all 462 possibilities in the sense of most closely balancing the total populations on the two mountains. Two distinctly

different possibilities have been advanced by the commentaries as to which division of the Levites fits into the Talmudic description of those who are fit to work in the temple, those between the ages of 30 and 50 those belonging to the Kehat family. Both of these explanations are in harmony with a mathematical division of the tribes so as to promote numerical equality. No other explanation (Talmudic or other) achieves this harmony.

If, instead of adopting the division we propose, all 23,000 of the counted Levites are assumed to have stood on Grizim, as some have interpreted the text, the population on Grizim exceeds that on Aval by 8,870, and it can be empirically determined that under that assumption, 51 other allocations would more closely equalize the population groups than would the biblical allocation. On the other hand, if all 23,000 Levites are assumed to have stood below, with none on Grizim, then the population on Grizim would be 14,130 fewer than the population on Aval, and 54 other allocations would more closely equalize the population groups. Only by dividing the Levites into between 8,770 and 8,970 in the valley, and between 14,030 and 14,230 on Grizim, does the biblical allocation optimally balance the two population groups. Just such a division seems to follow from the Talmudic opinion that those Levites worthy of serving in the Tabernacle were in the valley.

## CONCLUSION

Thus, our understanding of the rationale for dividing the tribes such that those who worked in the Tabernacle stood in the valley solves both[9] of the posed problems. It explains the biblical assignment of the tribes to the two mountains in the sequence found in Deuteronomy 27:11-12 as the most numerically balanced division of the tribes into two groups of six. It also harmonizes the assignment of some Levites to Grizim, with the special role of "calling out" performed by a special subset of Levites. The linking of the Levites with the Priests in the context of the description of the ceremony in the Book of Joshua further suggest that it was just those Levites who served together with the Priests in the Tabernacle who stood between the two mountains. What is interesting about our solution is that both of the understandings of the Talmudic phrase "those who worked in the Tabernacle"—those between 30 and 50 or the family of Kehat—prove mathematically optimal and no other solution does.[10]

The explanation we propose is the only one we are aware of which links both[11] of the difficulties found in Deuteronomy 27:11-14 and provides a single solution.

## POSTSCRIPT

Maimonides, writing in the *Laws of Repentance* (3:4), states:

> It is incumbent upon every person throughout the year to view himself as half innocent and half guilty. So too all of the world should be viewed as half innocent and half guilty. Every individual sin can determine his own—and the world's—fate for guilt and destruction. One good deed can place oneself—and the world—on the side of salvation and redemption.

Perhaps the division of the tribes on Aval and Grizim stands for just this proposition; the forces of truth and evil are equally balanced, and the actions of each individual can have an impact on the balance.

## NOTES

1. Nearly all the post-Talmudic commentators favor solutions based on dividing the Levites into two different groups, as that seems to better resolve the textual difficulties; See commentaries on Deuteronomy 27:12 of Rashi (as explained by *Sifte Hakhamim*); Rabbenu Bahya; *Hizkuni*; Rabbi S.R. Hirsch; Commentaries of Radak and *Metzudat David* on Joshua 8:33.

2. This assumes that sequence of the items in any given group as well as the mirror image of a group do not count. This is mathematically expressed as:

$$\text{Number of possibilities} = \frac{K!}{(K-M)! * M} * (.5)$$

with:

K = members of group (in our case 12).

M = number of members in each subunit (in our case 6).

! = factorial.

* = multiplication.

Thus:

$$\frac{12!}{(12-6)! * (6)!} * (.5) = 462$$

3. Rabbi Hizkiya ben Manoah (*Hizkuni*) states that the tribes were divided according to their military strength and the applicability of the curses to specific tribes. Rabbenu Bahya suggests that the tribes were divided by maternal linage with two tribes transferred to balance the division. Rabbi S.R. Hirsch adopts an explanation which is a hybrid of reason two above and Rabbenu Bahya.

4. Our analysis was performed with the aid of a computer program. We illustrate our findings in greater detail in the appendix.

5. See Numbers 4. After the first year in the desert, Levites aged 30 to 50 are numbered at 8,580, while the total number of Levites counted there is 22,300. The 30 to 50 age group thus comprised 38.48% of the total.

6. 38.48% of the Levites at that time which we are told in Numbers 26 is 23,000.

7. See Numbers 3:28, 3:39 (and Rashi on 3:39).

8. Other divisions of the tribe of Levi have been suggested to address the dual role played by the Levites in the ceremony; see section I. We have examined all of the possible divisions of the tribe of Levi that can be textually supported (i.e., based on divisions of the Levites found in the Bible), including, only Merari, only Gershon, and only priests as well as combinations of these groups (i.e. Gershon and Merari, Merari and priests and so on). None of these divisions fall within the numeric range which we have shown is necessary to make the biblical allocation optimal.

9. For reasons further explained in the postscript, our solution would also suggest why the six tribe group containing Shimon, Levi, Judah, Issachar, Yosef and Benjamin was assigned to Grizim as opposed to Aval. Being the more numerous of the two groups (at least according to Rashi's explanation), it would seem proper that it stand on the mountain that receives the blessings, rather than the curses, to demonstrate that while "The forces of truth and evil are equally balanced, and the actions of each individual can have an impact on the balance," truth will triumph.

   In addition, our explanation is compatible with the various other rationales advanced by the commentaries as to why this group stood in Grizim; see Commentaries of Rabbenu Bahya, *Hizkuni* and Rabbi S. R. Hirsch on Deuteronomy 27:12.

10. Rashi's explanation of the division, that those between the ages of 30 and 50 stood in the valley, seems to be more logical than Maharsha's as according to Maharsha the group in the valley would include all of Kehat from one month and older. It would seem more appropriate to limit those calling out to adults.

11. Rabbi David Frankel writing in the *Korban Ha'Eda*, his commentary to the Jerusalem Talmud (*Sotah 7:4*, s.v. *veyisrael mekorai*) advances an interpretation which is sufficiently similar to our explanation that it bears quoting. He states:

     > The Bible divided the tribes so that they would be nearly equal. Those on mount Grizim according to the census in [Numbers 27] were 302, 308 excluding the Levites older than 50 or younger than 30. Those on Mount Aval equaled 308,130. We find thus that those on Aval were more numerous than those on Grizim by 5,822. However, the division was near to equal.

This solution has within it certain key elements of our theory, most significantly that the tribes were divided so as to promote mathematical equality. However it lacks a number of critical components. First, the mathematical element of his explanation appears incorrect. We are unable to explain the origins of either the number 308,130 or 302,308. Neither number reflects a mathematical summation of the tribes on the mountains. Second, he seems to adopt the theory that the Levites *over* 50 and *under* 30 were in the valley, an explanation not found in the Talmud or commentaries. Finally, and most significantly, he does not relate the proper division of the Levites (in our opinion, 30 to 50 in the valley) with any division of the tribes that not only promotes equality, but is completely optimal. The difference of 5,822 he presents would be the 34th most optimal division.

In essence Rabbi Frankel grasped only half the solution, albeit a significant half. He realized that the tribes were divided to promote equality. He does not, however, realize that the same is true for the Levites, and that such a division not only promotes equality, but is completely optimal. Had Rabbi Frankel lived in our technologically advanced era he quite possibly would have refined his mathematical and textual analysis and grasped that the proper division of the tribes and Levites not only encourages equality, but creates optimal equality.

Tosaphot (Babylonian Talmud, *Sotah* 36A s.v. *mai vehetzyav*) while explaining the same text of the Jerusalem Talmud, states that in order for the tribes to be mathematically equal 8,870 Levites must be in the valley. Tosaphot does not, however, explain the division of the tribes with reference to the division of the Levites. Quite to the contrary, Tosaphot adopts an explanation of the Jerusalem Talmud designed to create a numerical *imbalance* between the tribes on the mountains.

# APPENDIX

This is a list of the 462 possible tribal combinations on Grizim and Aval using the tribal population totals found in the chart on page 51—(with Levy having a tribal population of 14,150). Each listing contains two 6 letter series and a number. The first six letter series (column A) indicates one potential group of six tribes. The second six letter series (column B) indicates the tribes that would be located on the other mountain using this division. The number in the third column (column C) is the numerical difference

in total population of the tribes between the group in the first column and the group in the second. The first column always contains the larger group of tribes. For example: the first listing in the first column is SLJIYB, which indicates Shimon, Levi, Yehudah, Issachar, Yosef and Binyamin. Their tribal population was 307,950. RDNGAZ, on the other hand, indicates Reuven, Gad, Dan, Naphtali Asher, and Zevulun, whose tribal population was 307,930. Since 307,950 - 307,930 = 20, the third column contains the number 20. Smaller numbers indicate more balanced populations. The total six tribe population for any given division in the appendix can be determined through the following equation: for tribes in the first column, $X - (Y * .5) = 307,940$; for tribes in the second column $X + (Y * .5) = 307,940$. $Y$ = number is column three; $X$ is tribal population.

| A | B | C | A | B | C | A | B | C | A | B | C | A | B | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SLJIYB | RDNGAZ | 20 | RLJGAY | SDNIZB | 11,080 | LDNIYB | RSJGAZ | 22,220 | LJDIZB | RSNGAY | 35,020 | RLDIZY | SJNGAB | 48,680 |
| RGAIZB | SLJDNY | 180 | RLIZYB | SJDNGA | 11,080 | RJNGAZ | SLDIYB | 24,180 | RLJNZY | SDGAIB | 35,060 | RSJGIY | LDNAZB | 48,980 |
| LNAIYB | RSJDGZ | 220 | SJDGIB | RLNAZY | 11,120 | RJGAZB | SLDNIY | 24,580 | RLJZYB | SDNGAI | 35,480 | RNGAIY | SLJDZB | 49,180 |
| SLJDYB | RNGAIZ | 220 | RSJNGY | LDAIZB | 11,180 | LJNAYB | RSDGIZ | 24,620 | JNGAIB | RSLDZY | 35,520 | RSJDGY | LNAIZB | 49,180 |
| RDGAZB | SLJNIY | 380 | RLDZYB | SJNGAI | 11,280 | RSDGIY | LJNAZB | 24,780 | JDNGAB | RSLIZY | 35,720 | RDNGAY | SLJIZB | 49,380 |
| LDNAYB | RSJGIZ | 420 | DNGAIB | RSLJZY | 11,320 | LJDGIZ | RSNAYB | 24,820 | LJNGIY | RSDAZB | 36,220 | RGAIYB | SLJDNZ | 49,580 |
| RLGIZY | SJDNAB | 880 | RSJGYB | LDNAIZ | 11,580 | RLJGZY | SDNAIB | 25,280 | SGAIZY | RLJDNB | 36,320 | JNGIZB | RSLDAY | 49,720 |
| RLJDNI | SGAZYB | 1,080 | RNGAYB | SLJDIZ | 11,780 | RSJAIZ | LDNGYB | 25,380 | LJDNGY | RSAIZB | 36,420 | RDGAYB | SLJNIZ | 49,780 |
| RLDGZY | SJNAIB | 1,080 | LONGIY | RSJAZB | 12,020 | DNGIZB | RSLJAY | 25,520 | SJDNAI | RLGZYB | 36,520 | JDNGZB | RSLAIY | 49,920 |
| RSDAIZ | LJNGYB | 1,180 | LDGIYB | RSJNAZ | 12,420 | RSJDAZ | LNGIYB | 25,580 | SDGAZY | RLJNIB | 36,620 | RSDAIY | LJNGZB | 50,580 |
| RLJDIB | SNGAZY | 1,480 | RSNAIY | LJDGZB | 12,580 | RNGZYB | SLJDAI | 25,980 | LJGIYB | RSDNAZ | 36,620 | LJDAIZ | RSNGYB | 50,620 |
| RSGAIY | LJDNZB | 2,780 | LJNAIZ | RSDGYB | 12,780 | RLAIZY | SJDNGB | 26,680 | LJDGYB | RSNAIZ | 36,920 | SJDNIZ | RLGAYB | 50,720 |
| LJGAIZ | RSDNYB | 2,820 | RSDNAY | LJGIZB | 12,780 | SJDGAI | RLNZYB | 26,720 | SJDAIB | RLNGZY | 36,920 | RLJAZY | SDNGIB | 51,080 |
| SJNGIZ | RLDAYB | 2,920 | LJDNAZ | RSGIYB | 12,820 | RSNIZY | LJDGAB | 26,780 | RSJNAY | LDGIZB | 36,980 | SJDIZB | RLNGAY | 51,120 |
| RSDGAY | LJNIZB | 2,980 | RSAIYB | LJDNGZ | 12,900 | RLDAZY | SJNGIB | 26,890 | RSJAYB | LDNGIZ | 37,380 | RSJNZY | LDGAIB | 51,180 |
| LJDGAZ | RSNIYB | 3,020 | LJAIZB | RSDNGY | 13,020 | RSDNZY | LJGAIB | 26,980 | SLJDIY | RNGAZB | 37,620 | DNAIZB | RSLJGY | 51,320 |
| SJDNGZ | RLAIYB | 3,120 | SJNIZB | RLOGAY | 13,120 | RSJGAY | LDNIZB | 27,180 | RDGAIZ | SLJNYB | 37,780 | RSJZYB | LDNGAI | 51,580 |
| SJGIZB | RLDNAY | 3,320 | RSDAYB | LJNGIZ | 13,190 | RSIZYB | LJDNGA | 27,190 | LDNAIY | RSJGZB | 37,820 | RNAZYB | SLJOGI | 51,780 |
| NGAIZB | RSLJDY | 3,520 | LJDAZB | RSNGIY | 13,220 | RSDZYB | LJNGAI | 27,390 | LDAIYB | RSJNGZ | 38,220 | LDNIZY | RSJGAB | 52,020 |
| SJDGZB | RLNAIY | 3,620 | SJDNZB | RLGAIY | 13,320 | JNGAZB | RSLDIY | 27,920 | SDNIYB | RLJGAZ | 38,320 | LJGAIY | RSDNZB | 52,220 |
| DNGAZB | RSLJIY | 3,720 | LNIZYB | RSJDGA | 14,420 | LDGAIY | RSJNZB | 28,020 | LJNZYB | RSDGAI | 38,820 | SJNGIY | RLDAZB | 52,320 |
| LNGIZY | RSJDAB | 4,220 | LJNGAY | RSDIZB | 14,420 | SDNGIY | RLJAZB | 28,120 | SJNAYB | RLDGIZ | 40,720 | LJDGAY | RSNIZB | 52,420 |
| LDNGZY | RSJAIB | 4,420 | LDNZYB | RSJGAI | 14,620 | SDGIYB | RLJNAZ | 28,620 | SJDGIZ | RLNAYB | 40,920 | LDIZYB | RSJNGA | 52,420 |
| SDNAIZ | RLJGYB | 4,520 | LJGAYB | RSDNIZ | 14,620 | LJNGZY | RSDAIB | 28,620 | DNGAIZ | RSLJYB | 41,120 | SJDNGY | RLAIZB | 52,520 |
| LGIZYB | RSJDNA | 4,620 | SJNGYB | RLDAIZ | 14,920 | SJNAIZ | RLDGYB | 28,720 | RSJGZY | LDNAIB | 41,380 | SJGIYB | RLDNAZ | 52,720 |
| LDGZYB | RSJNAI | 4,820 | SLJAIY | RDNGZB | 15,620 | SJDNAZ | RLGIYB | 28,920 | DGAIZB | RSLJNY | 41,520 | NGAIYB | RSLJDZ | 52,920 |
| LJDNIB | RSGAZY | 4,820 | SLJDAY | RNGIZB | 15,820 | LJGZYB | RSDNAI | 29,020 | RNGAZY | SLJDIB | 41,580 | SJDGYB | RLNAIZ | 52,920 |
| SDAIZB | RLJNGY | 4,920 | RJNGIB | SLDAZY | 16,160 | SJAIZB | RLDNGY | 29,120 | RGAZYB | SLJDNI | 41,980 | DNGAYB | RSLJIZ | 53,120 |
| RSNAZY | LJDGIB | 4,980 | SNAIYB | RLJDGZ | 16,320 | SJDAZB | RLNGIY | 29,320 | RJNAIB | SLDGZY | 41,980 | RJDNGI | SLAZYB | 53,780 |
| RLJNYB | SDGAIZ | 5,280 | RJDNGB | SLAIZY | 16,390 | SLJIZY | RDNGAB | 29,820 | RJDNAB | SLGIZY | 42,180 | SDNAIY | RLJGZB | 53,920 |
| RSAZYB | LJDNGI | 5,380 | SDNAYB | RLJGIZ | 16,520 | SLJDZY | RNGAIB | 30,020 | LDGIZY | RSJNAB | 42,220 | RJDGIB | SLNAZY | 54,180 |
| SLDIZY | RJNGAB | 5,620 | RSGIZY | LJDNAB | 16,980 | LNAIZY | RSJDGB | 30,020 | RLJNIY | SDGAZB | 42,680 | SDAIYB | RLJNGZ | 54,320 |
| SNGAIY | RLJDZB | 6,120 | RLJDAI | SNGZYB | 17,080 | LDNAZY | RSJGIB | 30,220 | RSAIZY | LJDNGB | 42,780 | LJNAZY | RSDGIB | 54,420 |
| SDNGAY | RLJIZB | 6,320 | RSDGZY | LJNAIB | 17,180 | LAIZYB | RSJDNG | 30,420 | RLJDNY | SGAIZB | 42,880 | LJAZYB | RSDNGI | 54,820 |
| SGAIYB | RLJDNZ | 6,520 | RSJDNI | LGAZYB | 17,180 | SJNGAY | RLDIZB | 30,520 | RSDAZY | LJNGIB | 42,980 | SJNZYB | RLDGAI | 54,920 |
| SDGAYB | RLJNIZ | 6,720 | RSJDIB | LNGAZY | 17,580 | SNIZYB | RLJDGA | 30,520 | RLJIYB | SDNGAZ | 43,080 | RJNIZB | SLDGAY | 55,180 |
| RSJDGI | LNAZYB | 7,380 | RDNAIB | SLJGZY | 17,780 | LDAZYB | RSJNGI | 30,620 | RLJDYB | SNGAIZ | 43,280 | RJDNZB | SLGAIY | 55,380 |
| RDNGAI | 6LJZYB | 7,580 | RLDNIY | SJGAZB | 18,480 | SDNZYB | RLJGAI | 30,720 | SDGAIY | RLJNZB | 44,120 | JDNGIB | RSLAZY | 57,520 |
| RDGAIB | SLJNZY | 7,990 | RLDIYB | SJNGAZ | 18,680 | SJGAYB | RLDNIZ | 30,920 | LJGAZY | RSDNIB | 44,620 | RJNGYB | SLDAIZ | 57,980 |
| SLJAZY | RDNGIB | 8,020 | SJGAIZ | RLDNYB | 18,920 | RLJDIZ | SNGAYB | 31,280 | SJNGZY | RLDAIB | 44,720 | SDGIZY | RLJNAB | 58,320 |
| RJNGZB | SLDAIY | 8,580 | SJDGAZ | RLNIYB | 19,120 | RJNGAI | SLDZYB | 31,780 | SJGZYB | RLDNAI | 45,120 | RLJAIY | SDNGZB | 58,680 |
| RLOGIY | SJNAZB | 8,680 | LGAIZY | RSJDNB | 20,220 | RJDNGA | SLIZYB | 31,980 | NGAZYB | RSLJDI | 45,320 | RSJNIY | LDGAZB | 58,780 |
| SNAZYB | RLJDGI | 9,720 | SNGIZY | RLJDAB | 20,320 | RDNIZB | SLJGAY | 31,980 | RLNGIZ | SLDAYB | 45,880 | RLJDAY | SNGIZB | 58,980 |
| RLJAIZ | SDNGYB | 9,280 | LJDNAI | RSGZYB | 20,420 | RJGAIB | SLDNZY | 32,180 | SNAIZY | RLJDGB | 46,120 | RSJDNY | LGAIZB | 58,980 |
| RSJNIZ | LDGAYB | 9,380 | LDGAZY | RSJNIB | 20,420 | RJDGAB | SLNIZY | 32,380 | RJDNGZ | SLAIYB | 46,180 | RSJIYB | LDNGAZ | 59,180 |
| RLJDAZ | SNGIYB | 9,480 | SDNGZY | RLJAIB | 20,620 | RLJGIY | SDNAZB | 32,680 | SDNAZY | RLJGIB | 46,320 | RNAIYB | SLJDGZ | 59,380 |
| RSJDNZ | LGAIYB | 9,640 | SGIZYB | HLJDNA | 20,720 | RLJDGY | GNAIZB | 33,080 | RJGIZB | SLDNAY | 46,380 | RSJDYB | LNGAIZ | 59,300 |
| RGJIZB | LDNGAY | 9,790 | LJDAIB | RSNGZY | 20,820 | RSJNAI | LDGZYB | 33,180 | LJNIYB | RSDGAZ | 46,420 | RDNAYD | OLJOIZ | 59,580 |
| RNAIZB | SLJDGY | 9,980 | RLJNAY | SDGIZB | 20,880 | RNGIYB | SLJDAZ | 33,580 | SAIZYB | RLJDNG | 46,520 | SJGAZY | RLDNIB | 60,720 |
| RSJDZB | LNGAIY | 9,980 | SDGZYB | RLJNAI | 20,920 | RDNGYB | SLJAIZ | 33,780 | RJDGZB | SLNAIY | 46,580 | RJGAIZ | SLDNYB | 61,900 |
| RDNAZB | SLJGIY | 10,180 | SJDNIB | RLGAZY | 20,920 | RJNAZB | SLDGIY | 34,380 | LJDNYB | RSGAIZ | 46,620 | LJNAIY | RSDGZB | 62,020 |
| LJDGAJ | RSNZYB | 10,620 | RLJAYB | SDNGIZ | 21,280 | RLDAIY | SJNGZB | 34,480 | SDAZYB | RLJNGI | 46,720 | RJDGAZ | SLNIYB | 62,180 |
| RLNIZY | SJDGAB | 10,680 | RSJNYB | LDGAIZ | 21,380 | RSDNIY | LJGAZB | 34,580 | RSJDIZ | LNGAYB | 47,380 | LJDNAY | RSGIZB | 62,220 |
| SJDNGI | RLAZYB | 10,720 | RDNGIZ | SLJAYB | 21,780 | LJDNIZ | RSGAYB | 34,620 | RDNAIZ | SLJGYB | 47,580 | LJAIYB | RSDNGZ | 62,420 |
| RLDNZY | SJGAIB | 10,980 | RDGIZB | SLJNAY | 22,180 | RSDIYB | LJNGAZ | 34,980 | RDAIZB | SLJNGY | 47,980 | SJNIYB | RLDGAZ | 62,520 |

| A | B | C | A | B | C | A | B | C | A | B | C | A | B | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LJDAYB | RSNGIZ | 62,620 | RDNZYB | SLJGAI | 73,780 | RJNGZY | SLDAIB | 87,780 | JDGAIZ | RSLNYB | 103,320 | JDNGZY | RSLAIB | 129,120 |
| SJDNYB | RLGAIZ | 62,720 | RJGAYB | SLDNIZ | 73,980 | RJGZYB | SLDNAI | 88,180 | RJGAZY | SLDNIB | 103,780 | JGIZYB | RSLDNA | 129,320 |
| RNGIZY | SLJDAB | 63,380 | LJDGIY | RSNAZB | 74,220 | RSJIZY | LDNGAB | 88,980 | DNGIZY | RSLJAB | 104,720 | JDGZYB | RSLNAI | 129,520 |
| RDNGZY | SLJAIB | 63,580 | RSJAIY | LDNGZB | 74,780 | RSJDZY | LNGAIB | 89,180 | DGIZYB | RSLJNA | 105,120 | SJDIZY | RLNGAB | 130,320 |
| RGIZYB | SLJDNA | 63,780 | DNGIYB | RSLJAZ | 74,920 | RNAIZY | SLJDGB | 89,180 | RJNIYB | SLDGAZ | 105,580 | DNAIZY | RSLJGB | 130,520 |
| RJDNIB | SLGA2Y | 63,980 | RSJDAY | LNGIZB | 74,980 | RDNAZY | SLJGIB | 89,380 | RJDNYB | SLGAIZ | 105,780 | DAIZYB | RSLJNG | 130,920 |
| RDGZYB | SLJNAI | 63,990 | JNAIZB | RSLDGY | 75,520 | RAIZYB | SLJDNG | 89,580 | JNGAZY | RSLDIB | 107,120 | RJDGIY | SLNAZB | 133,380 |
| RSDIZY | LJNGAB | 64,780 | JDNAZB | RSLGIY | 75,720 | RDAZYB | SLJNGI | 89,780 | JGAZYB | RSLDNI | 107,520 | RJNIZY | SLDGAB | 135,380 |
| JNGAIZ | RSLDYB | 65,320 | LJNIZY | RSDGAB | 76,220 | SJDGIY | RLNAZB | 90,320 | SJAIZY | RLDNGB | 108,320 | RJDNZY | SLGAIB | 135,580 |
| JDNGAZ | RSLIYB | 65,520 | LJDNZY | RSGAIB | 76,420 | DNGAIY | RSLJZB | 90,520 | SJDAZY | RLNGIB | 108,520 | RJIZYB | SLDNGA | 135,780 |
| JGAIZB | RSLDNY | 65,720 | LJIZYB | RSDNGA | 76,620 | DGAIYB | RSLJNZ | 90,920 | RJDAIZ | SLNGYB | 109,780 | RJDZYB | SLNGAI | 135,980 |
| JDGAZB | RSLNIY | 65,920 | LJDZYB | RSNGAI | 76,820 | JNGZYB | RSLDAI | 91,520 | RDNIZY | SLJGAB | 111,180 | JDNGIY | RSLAZB | 136,720 |
| LJGIZY | RSDNAB | 66,420 | JNGAYB | RSLDIZ | 77,320 | LJAIZY | RSDNGB | 92,220 | RJGAIY | SLDNZB | 111,380 | JDGIYB | RSLNAZ | 137,120 |
| LJDGZY | RSNAIB | 66,620 | SJNAIY | RLOGZB | 78,120 | SJNIZY | RLDGAB | 92,320 | RJDGAY | SLNIZB | 111,580 | JNIZYB | RSLDGA | 139,120 |
| SJDAIZ | RLNGYB | 66,720 | SJDNAY | RLGIZB | 78,320 | LJDAZY | RSNGIB | 92,420 | RDIZYB | SLJNGA | 111,580 | JDN?YB | RSLGAI | 139,320 |
| NGIZYB | RSLJDA | 67,120 | SJAIYB | RLDNGZ | 78,520 | SJDNZY | RLGAIB | 92,520 | JDNAIZ | RSLGYB | 113,120 | RJDNIY | SLGAZB | 143,180 |
| RSJAZY | LDNGIB | 67,180 | SJDAYB | RLNGIZ | 78,720 | SJIZYB | RLDNGA | 92,720 | JDAIZB | RSLNGY | 113,520 | RJDIYB | SLNGAZ | 143,580 |
| DNGZYB | RSLJAI | 67,320 | RGAIZY | SLJDNB | 79,360 | SJDZYB | RLNGAI | 92,920 | RJNAZY | SLDGIB | 113,580 | JGAIZY | RSLDNB | 144,920 |
| LDAIZY | RSJNGB | 68,020 | RDGAZY | SLJNIB | 79,560 | NAIZYB | RSLJDG | 92,920 | RJAZYB | SLDNGI | 113,980 | JDGAZY | RSLNIB | 145,120 |
| SDNIZY | RLJGAB | 68,120 | RJDNAI | SLGZYB | 79,560 | DNAZYB | RSLJGI | 93,120 | LJDIZY | RSNGAB | 114,220 | JDNIYB | RSLGAZ | 146,920 |
| SJGAIY | RLDNZB | 68,320 | RJDAIB | SLNGZY | 79,980 | RJDNIZ | SLGAYB | 93,780 | JNGAIY | RSLDZB | 114,720 | RJAIZY | SLDNGB | 151,380 |
| SDIZYB | RLJNGA | 68,520 | RLJDIY | SNGAZB | 80,680 | RJDIZB | SLNGAY | 94,180 | DNIZYB | RSLJGA | 114,920 | RJDAZY | SLNGIB | 151,580 |
| SJDGAY | RLNIZB | 68,520 | RDNIYB | SLJGAZ | 81,380 | RJNGIY | SLDAZB | 95,380 | JDNGAY | RSLIZB | 114,920 | JDGAIY | RSLNZB | 152,720 |
| RJDGAI | SLNZYB | 69,780 | SJGIZY | RLDNAB | 82,520 | RJDNGY | SLAIZB | 95,580 | JGAIYB | RSLDNZ | 116,120 | JNAIZY | RSLDGB | 154,720 |
| SJNAZY | RLDGIB | 70,520 | NGAIZY | RSLJDB | 82,720 | RJGIYB | SLDNAZ | 95,780 | JDGAYB | RSLNIZ | 116,320 | JDNAZY | RSLGIB | 154,920 |
| SJAZYB | RLDNGI | 70,920 | SJDGZY | RLNAIB | 82,720 | RJDGYB | SLNAIZ | 95,980 | SJDAIY | RLNGZB | 116,120 | JAIZYB | RSLDNG | 155,120 |
| RDNGIY | SLJAZB | 71,180 | DNGAZY | RSLJIB | 82,920 | RSJDIY | LNGAZB | 96,780 | JNAZYB | RSLDGI | 117,320 | JDAZYB | RSLNGI | 155,320 |
| RDGIYB | SLJNAZ | 71,580 | GAIZYB | RSLJDN | 83,120 | RDNAIY | SLJGZB | 96,990 | DGAIZY | RSLJNB | 120,720 | RJDAIY | SLNGZB | 159,180 |
| RJNAIZ | SLDGYB | 71,780 | DGAZYB | RSLJNI | 83,320 | RDAIYB | SLJNGZ | 97,380 | RJNAIY | SLDGZB | 121,180 | JDNAIY | RSLGZB | 162,520 |
| RJDNAZ | SLGIYB | 71,980 | JDNAIB | RSLGZY | 83,320 | JDNIZB | RSLGAY | 97,520 | RJDNAY | SLGIZB | 121,380 | JDAIYB | RSLNGZ | 162,920 |
| RJAIZB | SLONGY | 72,180 | RJNAYB | SLDGIZ | 83,780 | RJNZYB | SLDGAI | 97,980 | RJAIYB | SLDNGZ | 121,580 | JDGIZY | RSLNAB | 166,920 |
| RJDAZB | SLNGIY | 72,380 | RJDGIZ | SLNAYB | 83,980 | JNGIYB | RSLDAZ | 99,120 | RJDAYB | SLNGIZ | 121,780 | RJDIZY | SLNGAB | 173,380 |
| RLJIZY | SDNGAB | 72,680 | LJDNIY | RSGAZB | 84,020 | JDNGYB | RSLAIZ | 99,320 | JNAIYB | RSLDGZ | 124,920 | JDNIZY | RSLGAB | 176,720 |
| RLJDZY | SNGAIB | 73,080 | SDAIZY | RLJNGB | 84,120 | LJDAIY | RSNGZB | 100,020 | JDNAYB | RSLGIZ | 126,120 | JDIZYB | RSLNGA | 177,120 |
| JDNGAI | RSLZYB | 73,120 | LJDIYB | RSNGAZ | 84,420 | SJDNIY | RLGAZB | 100,120 | RJGIZY | SLDNAB | 126,580 | JDAIZY | RSLNGB | 192,720 |
| JDGAIB | RSLNZY | 73,520 | RDGAIY | SLJNZB | 87,180 | SJDIYB | RLNGAZ | 100,620 | RJDGZY | SLNAIB | 126,780 | | | |
| PNIZYB | SLJDGA | 73,580 | JDNGIZ | RSLAYB | 87,320 | DNAIYB | RSLJGZ | 100,720 | RDAIZY | SLJNGB | 127,180 | | | |
| RJNGAY | SLDIZB | 73,580 | JDGIZB | RSLNAY | 87,720 | RDGIZY | SLJNAB | 101,380 | JNGIZY | RSLDAB | 128,920 | | | |