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THE DIVISION OF THE TRIBES ON GERIZIM AND EVAL, OR THE SOLUTION TO DEUTERONOMY IS NOT IN NUMBERS

INTRODUCTION

Prior to their entry into *Erets Yisrael*, the Israelites were told that they were masters of their own fate.¹ If they chose to follow the Torah and its laws they would be blessed, and if not they would be cursed. They were also told that a formal ceremony memorializing this covenant would take place upon their entering the land, on the mountains of Gerizim and Eval.²

In describing the ceremony, the Torah lists the twelve tribes that were to represent the Israelites, and specifies how these tribes were to be deployed between the two mountains. The reason for the specific division is not stated, nor does the division resemble any other found elsewhere in the Torah. Both the Jerusalem and Babylonian Talmuds discuss the ceremony in great detail, but neither offers a plausible reason for the specified division.

Michael J. Broyde and Steven S. Weiner (*Tradition* 27:1, Fall 1993)³ offer a mathematical solution to the problem. They suggest that the division of the twelve tribes between the two mountains was intended to equalize the number of people on each mountain. Based on an analysis of the two census counts in *BaMidbar*, they project and extrapolate the population size of each of the tribes at the time of entry into *Erets Yisrael*. They conclude that the division of the tribes listed in the Torah

results in a difference of between 1 and 20 people between the two mountains, and that all other divisions of the twelve tribes into two groups of six yield considerably greater differences.

Broyde and Weiner's analysis follows the recent popular trend to turn to science and computer-based numerical and statistical calculations to explain and confirm the Torah and its teachings. More and more these attempts take the form of showing the Torah's precision and exactitude. However, regardless of how striking the numerical results may be, and in this case they are extremely impressive, the ultimate test of the correctness of a theory lies not in its numerical exactitude but on the soundness of its logic. In this paper we carefully review the Broyde and Weiner analysis and demonstrate that their underlying assumptions are in all likelihood incorrect, and their numerical extrapolations are unconvincing.

In the concluding section of the paper, we offer a non-mathematical geographic explanation for the division. We show that our explanation is consistent with another tribal division found in the Torah and that unlike Broyde and Weiner's solution, is consistent with all of the biblical and talmudic sources.

WHO STOOD WHERE: THE TORAH AND THE TALMUD

At the ceremony inaugurating their entry into *Erets Yisrael*, the Torah commands the Israelites to divide twelve tribes on the mountains of Gerizim and Eval according to the following assignment:

Mount Gerizim: Shimon, Levi,⁴ Yehuda, Yissakhar, Yosef and Binyamin.

Mount Eval: Reuven, Gad, Asher, Zevulun, Dan and Naftali.

Verse 14 then adds: "And the Levites answered, and they spoke unto every man in Israel in a loud voice."

The task assigned to these Levites was to sequentially announce a series of blessings and curses. The blessings would follow if the Israelites obeyed the commandments of the Torah, the curses if they did not.

The instructions, as listed, are incomplete. Firstly, no mention is made as to where the Levites calling out the blessings and curses were to stand. Secondly, since the tribe of Levi had been instructed to be on Mount Gerizim it is unclear as to which Levites the Torah is referring to in verse 14. Some clarification on both of these questions is given in

Joshua 8:33: “And all of Israel and its elders and judges are standing on both sides of the ark facing the Levite Priests, the carriers of the ark of God’s covenant (*nose’ei aron berit Hashem*), both converts and inhabitants, half of them facing Mount Gerizim and half of them facing Mount Eval, in the manner in which God had commanded Moshe the servant of God to first bless the nation Israel.”

The Babylonian Talmud (*Sota* 32a and 36a-37a) offers three explanations of the above verses:

- 1) All people, Levites and Israelites, were in the valley;
- 2) The elders of the Priests and Levites were in the valley and all the rest were on the mountain;
- 3) The Levites who were involved in the work of the *Mishkan* were in the valley, while all other Levites were on the mountain, i.e. the Levites in the valley,
 - a) according to Rashi, were all Levites between the ages of 30 and 50 who worked in the *Mishkan*;
 - b) according to Maharsha, were of the clan of Kehat, who had the task of carrying the ark.

The Jerusalem Talmud (*Sota* 7:4), offers another possibility:

- 4) *HaKohanim haLevi'im* refers to Priests from the tribe of Levi,⁵ i.e., only the Priests, not the Levites, were in the valley.

In addition to where the Levites stood, the Babylonian Talmud (*Sota* 36a-b) and Jerusalem Talmud (*Sota* 7:4) also discuss the manner in which the twelve tribes were divided into two groups. The particular tribal partition is found nowhere else in the Pentateuch, and does not follow any age, common mother or *degel* pattern. The Talmud offers no intuitively reasonable solution for the selection of this partition.^{6,7}

A MATHEMATICAL SOLUTION

Broyde and Weiner suggest that the objective of the tribal division was to equalize, as best as possible, the number of *males* on each mountain. In total, there are 462 possible ways of partitioning twelve tribes into two groups of six.⁸ They calculate the cumulative male population for each of the 462 partitions, and show that for the Babylonian Talmud’s third possible explanation, the difference between the cumulative populations on Gerizim and Eval: according to Rashi, is 20; according to Maharsha, is 1. All of the other 461 tribal partitions yield larger differences.

Because not all of the data necessary for their computations are given by the Torah, Broyde and Weiner employ certain extrapolations and assumptions to arrive at their final figures. We will briefly summarize their analysis.

Our knowledge of the population of each tribe during the wandering of the Israelites in the *midbar* is based on two censuses recorded in *Numbers*. The first census⁹ was taken a year after the exodus from Egypt and supplies data on:

—the male population of each tribe over the age of 20 (with the exception of the tribe of Levi),

—the male population of each of the three Levite clans over the age of 30 days, and

—the male population of each of the three Levite clans between the ages of 30 and 50 years.

The second census¹⁰ was taken when the Israelites were on the verge of entering the Holy Land and supplies data on:

—the male population of each tribe over the age of 20 (with the exception of the tribe of Levi), and

—the *total* male Levite population over 30 days of age.¹¹

For the purposes of Broyde and Weiner's calculations, neither census is adequate. The first lacks timeliness, since it was taken 39 years prior to the entry into *Erets Yisrael*. The second, though timely, lacks critical population data on the Levite sub-classes.¹² The authors' solution is to use the second census and project the Levite sub-class populations by assuming that their proportion of the total male Levite population remained unchanged in the 39 year interim between the first and second census.

In the first census there were a total of 22,300 male Levites over 30 days of age.¹³ Of these 8,580 (38.48%) were between the ages of 30 and 50,¹⁴ and 8,600 (38.57%) were of the clan of Kehat over 30 days of age.¹⁵ Since the total male Levite population in the second census was 23,000,¹⁶ Broyde and Weiner therefore estimate that when the Israelites entered *Erets Yisrael* there were 8,850 male Levites between the ages of 30 and 50 (i.e., $.3848 \cdot 23,000$) and 8,871 males in the clan of Kehat over the age of 30 days (i.e., $.3857 \cdot 23,000$).

They then calculate the populations on each mountain for the best partition (Table 1) as well as for all other partitions.

TRADITION

TABLE 1

Rashi				Maharsha			
<i>Gerizim</i>		<i>Eval</i>		<i>Gerizim</i>		<i>Eval</i>	
Shimon	22,200	Reuven	43,730	Shimon	22,200	Reuven	43,730
Levi*	14,150	Gad	40,500	Levi*	14,129	Gad	40,500
Yehuda	76,500	Asher	53,400	Yehuda	76,500	Asher	53,400
Yissakhar	64,300	Zevulun	60,500	Yissakhar	64,300	Zevulun	60,500
Yosef	85,200	Dan	64,400	Yosef	85,200	Dan	64,400
Binyamin	45,600	Naftali	45,400	Binyamin	45,600	Naftali	45,400
Total	307,950		307,930	Total	307,929		307,930

*Note: This is the number of Levites who were on the mountain. It is derived by subtracting the relevant number of Levites (8,850 for Rashi and 8,871 for Maharsha) from the 23,000 total number of male Levites over the age of 30 days.

Table 2 itemizes the second best partition. Note the population differences go from 20 and 1 in Table 1, to 180 and 201 in Table 2.

In a postscript, Broyde and Weiner offer a rationale for wanting cumulative equality on both mountains. They cite the well-known religious dictum that suggests that everyone view himself, as well as the entire world, as being in perfect equilibrium with respect to good and bad deeds.¹⁷ They argue that

Perhaps the division of the tribes on Aval and Gerizim stands for just this proposition; the forces of truth and evil are equally balanced, and the actions of each individual can have an impact on the balance.

Broyde and Weiner point out that their analysis works only according to the third explanation of the Babylonian Talmud as to where the Levites stood. For the other two explanations offered by the Babylonian Talmud and the explanation offered by the Jerusalem Talmud, they assert that the partition of the 12 tribes into the 2 groups of 6 given in the Torah can be improved on by 51 other partitions. For these other explanations the authors offer no rationale for why the given partition was chosen.

TABLE 2

Rashi				Maharsha			
<i>Gerizim</i>		<i>Eval</i>		<i>Gerizim</i>		<i>Eval</i>	
Shimon	22,200	Reuven	43,730	Shimon	22,200	Reuven	43,730
Levi	14,150	Gad	40,500	Levi	14,129	Gad	40,500
Yehuda	76,500	Asher	53,400	Yehuda	76,500	Asher	53,400
Dan	64,400	Zevulun	60,500	Dan	64,400	Zevulun	60,500
Yosef	85,200	Yissakhar	64,300	Yosef	85,200	Yissakhar	64,300
Naftali	45,400	Binyamin	45,600	Naftali	45,400	Binyamin	45,600
Total	307,850		308,030	Total	307,829		308,030

A CRITIQUE

The population differences that Broyde and Weiner arrive at, 20 and 1, are admittedly striking. However, it is highly unlikely that they are accurate. As explained in the previous section, these numbers assume that the relevant proportions of sub-classes of Levites remained exactly the same over a 40 year period. If these proportions change, even slightly, the population differences will increase. Moreover, Broyde and Weiner calculate that in order for the given tribal partition to continue to yield the minimum population differences, the estimated proportions of the Levite sub-classes must be between 38.2% and 39.0%. Hence, a shift in this ratio over a 40 year period of less than 1% would make some other partition optimal. Assuming that the proportion of all male Levites between the ages of 30 and 50, and the proportion of males over the age of 30 days for the clan of Kehat are both 38.0% (i.e. about 135 males less than the value calculated by Broyde and Weiner), Table 3 gives the cumulative populations on each mountain for the tribal partitions discussed in Tables 1 and 2. The partition in the Pentateuch is no more optimal.

TRADITION

TABLE 3

Partition in Pentateuch				Partition from Table 2			
<i>Gerizim</i>		<i>Eval</i>		<i>Gerizim</i>		<i>Eval</i>	
Shimon	22,200	Reuven	43,730	Shimon	22,200	Reuven	43,730
Levi	14,260	Gad	40,500	Levi	14,260	Gad	40,500
Yehuda	76,500	Asher	53,400	Yehuda	76,500	Asher	53,400
Yissakhar	64,300	Zevulun	60,500	Dan	64,400	Zevulun	60,500
Yosef	85,200	Dan	64,400	Yosef	85,200	Yissakhar	64,300
Binyamin	45,600	Naftali	45,400	Naftali	45,400	Binyamin	45,600
Total	308,060		307,930	Total	307,960		308,030

In considering the likelihood of proportional changes, we need only review the recorded population shifts that occurred during the 40 years that the Israelites wandered in the desert. A comparison of the population of the other tribes shows that the smallest percentage population shift for any tribe was 2.55% (Yehuda), and the largest was 63.6% (Menashe). Additionally, within the tribes a number of clans became extinct. Among these are at least two Levite families, including at least one associated with Kehat.¹⁸ In light of these population shifts, the proportional stability required by Broyde and Weiner is simply not present.

A second difficulty with the analysis¹⁹ is the assumption that all male Levites over the age of 30 days were on Gerizim,²⁰ while all of the other males from the other tribes were over 20 years of age. There is no immediately apparent reason for the Levite males and the males of the other tribes who were on the mountain to differ in age. The blessings and curses offered during the ceremony depend on the Torah observance of adults, not on the actions of minors.²¹ If we attempt to compensate for this modification by limiting our calculations to Levites over the age of 20, we are stymied by the fact that the Torah in neither census provides this information.

While the above critiques argue against taking the differences of 1 and 20 mentioned by Broyde and Weiner too seriously, they do not refute the basic thesis that equality of population was the desired objective. Thus, although we have no proof as to the exact differences of any of the partitions, the partition given in the Torah may very well have been optimal. For that matter any of the other two explanations of the Babylonian Talmud and the single explanation of the Jerusalem Talmud may also satisfy the requirements of optimal population equilibrium. Broyde and Weiner's assertion of 51 superior partitions for the other

explanations is based on counting all male Levites over the age of 30 days. However, as we have already argued, we have no reason to assume that this is the proper target group.²²

The difficulty with Broyde and Weiner's explanation, however, is not limited to the fact that it doesn't work according to 3 of the 4 talmudic explanations of which Levites stood on the mountain, and that it involves unmotivated assumptions and extrapolations. Even assuming their extrapolative assumptions and the necessary Babylonian Talmud interpretation of the location of the positioning of the Levites, Broyde and Weiner's analysis appears to be contradicted by a scriptural passage. According to *Joshua* (4:13), the tribes of Gad, Reuven and half of Menashe did not accompany the remaining tribes into *Erets Yisrael* but rather sent a contingent of 40,000 fighting men to assist the remaining tribes in their conquest of *Erets Yisrael*.²³ This means that in Table 1, both according to Rashi and Maharsha, the 84,320 men from the combined forces of Gad and Reuven that stood on Eval, overcounted the representation by at least 44,320 (i.e., 84,320-40,000). Thus, even if no Levites stood on Gerizim, the cumulative population of Eval was significantly smaller.²⁴ Surprisingly, Broyde and Weiner make no mention of this problem even though the argument is explicitly stated in the Jerusalem Talmud in the name of R. Yosi ben Bun as follows: "For no one came from the tribes of Reuven and Gad except for 40,000 soldiers."²⁵ This is also mentioned by *Tosafot* in *Sota* 36a, s.v. "*mai*." Although the Babylonian Talmud makes no overt statement on the subject, a careful reading of *Sota* 36b would indicate that it too agrees with the argument.

AN ALTERNATIVE EXPLANATION

Although Broyde and Weiner's objective of mathematical equilibrium is interesting, we would like to offer a more compelling non-mathematical explanation as to why the particular partition was selected. The *degalim*²⁶ partitioned the 12 tribes into 4 groups of 3. Mathematical population equilibrium was certainly not the objective in this situation, as the population of the *degalim* ranged from a low of 108,100 to a high of 186,400. Rather, as Ibn Ezra explains,²⁷ it was based on familial considerations. Ideally we would prefer that tribes of a common mother remain together. However, since Levi was not counted in the *degalim*, Leah was left with five tribes. Accordingly, the division consisted of all the children of Rachel being together, one group of 3 of Leah's chil-

dren being together, and the remaining two children of Leah, i.e., Reuven and Shimon being supplemented by Gad, the eldest son of Leah's maidservant.

We thus see that in partitioning the tribes the Torah gives credence to familial relationships. This does not mean that numerical equilibrium is not a desirable objective. Rather it argues that other alternate objectives may be equally important. It is certainly understandable that when it comes to living and being together on a permanent or regular basis family ties are important and cannot be ignored. A shared ancestry and heritage makes it likely that needs, desires and ambitions are closely related and allows for a more harmonious environment.

Seven years after entering Israel the land was divided amongst the tribes. Looking at the tribal partition of the Land, we note that all of the tribes on Gerizim lived in one contiguous geographic area in Israel. Starting in the south of Israel, they stretched directly north in a straight line uninterrupted by any of the other tribes.²⁸

Based on the above, we suggest that upon the Jews entering the land of Israel, the tribes were partitioned on the mountains for the blessing and cursing ceremony in such a way as to stress that the people's ability to follow the precepts of the Torah depended heavily on their environment, their immediate neighbors,²⁹ and their common ancestry. Whereas during their 40 years of wandering in the desert familial relationship took precedence, for the period of their residence in the Land of Israel and their eventual wandering in *galut* a common bond between neighbors would be essential for the survival of the people of Israel.

It is interesting to note that during Moshe's lifetime the ultimate division of the land amongst the tribes was not known. The individual tribal land assignments took place at the end of Joshua's reign using a lottery system. Moreover, the tribe of Dan was originally given land adjacent to the tribe of Efrayim. According to our explanation, Dan should have been on Gerizim. However, as mentioned in *Judges* 18:23, Dan was unable to conquer his allotted land and eventually left his assigned inheritance and took up residence in the north in Layash. Thus, since Dan in the ultimate scheme did not live near the other tribes he was not placed on Gerizim. Hence, the message of the Torah in choosing to partition the 12 tribes between the two mountains may very well have not been understood by the Israelites who actually stood on the mountains. It was only after more than a 100 years of post-Mosaic history and the final settling of the land that the message of the mountain partition could be appreciated.

Interestingly enough, our geographic-based solution for the tribal divisions does have a biblical precedence. When Moshe offers his farewell blessings to the tribes at the end of the Torah (*Devarim* 33), he starts by addressing them in the following order: Reuven, Yehuda, Levi, Binyamin, Yosef . . . Ramban explains (33:6) the pattern as follows:

After that he began with Yehuda, for he is the first inheritor in the Land and the leader of it, And he goes first to the battle regarding which he (Moshe) blesses him, and thus his blessing includes all Israel. Next he blessed Levi, who live adjacent to Yehuda in Jerusalem, where their sacrifices shall [hopefully] be accepted. He then blessed Binyamin, whose inheritance was adjacent to Yehuda. And the city of Jerusalem and the Temple were situated between the lands of Yehuda, Binyamin, and the Levites dwell among both of them. After this he blessed the children of Yosef, corresponding to their inheritance, as the Bible says: "And the inheritance of the standard of the children of Binyamin was chosen according to its families, and the border of their inheritance was situated between the children of Yehuda and the children of Joseph."

NOTES

1. *Devarim* 11:26-32.
2. *Ibid.* 27:11-26.
3. "A Mathematical Analysis of the Division of the Tribes and the Role of the Levites on Gerizim and Aval in Deuteronomy 27," *Tradition* 27:1, Fall 1992.
4. Note that Levi is treated as one of the tribes and that Yosef is not divided into the customary Menashe and Efrayim.
5. See *Bekhorot* 4a.
6. For example, the *gemara* suggests as a possible objective the equalization of the number of letters in the names of the tribes on either mountain (similar but different than the division on the *efod*). However, the *gemara* notes, this equality is realized only by non-standard spellings of the tribal names.
7. Some suggest that the division is according to greatness in and dedication to Torah. We may understand the inclusion of Shimon according to *Rashi Bereshit* 49:7: "Another approach is that there are no poor people, scribes and teachers of children except those of Shimon (See *Emet leYa'akov* on this *Rashi*).
8. I.e., $({}_{12}C_6)/2 = 462$. This is the general formula for combinations except for the fact that every time you take a group of 6 a mirror image, which is also used, is simultaneously being formed. Thus, in the end the number of possibilities is one half of ${}_{12}C_6$.
9. *BaMidbar* 1:20-47, 3:14-39, 4:34-48.

10. *BaMidbar* 26:1-62.
11. But not the population of the individual clans.
12. I.e., the total male Levite population between 30 and 50 years of age according to Rashi, and the total male clan of Kehat above the age of 30 days according to Maharsha.
13. The total of the three Levite clans as given in *BaMidbar* 3:22, 28, 34.
14. *BaMidbar* 4:48.
15. *BaMidbar* 3:28.
16. *BaMidbar* 26:62.
17. See for example *Mishne Torah Hilkhot Teshuva* 3:4.
18. See *Rashi Bamidbar* 26:58 s.v. "eile."
19. See *Rashash Sota* 36a, s.v. "va-afilu."
20. Regardless of which Levite group is being included.
21. Although a person becomes an adult at the age of 13, the census included only those over the age of 20. With respect to our current analysis, one could argue that 20 rather than 13 is used since *oneshin min ha-shamayim* apply over the age of 20. However, nowhere does it specifically say that the people on the mountains should be over the age of 20 (see *Rashash* footnote 19). If we assume that anyone over the age of 13 ascended the mountain, then the entire numerical analysis presented by the authors is wrong.
22. One could perhaps even salvage the basic premise of the Broyde and Weiner solution. According to their calculations, as long as the Levites on the mountain numbered between 14,030 and 14,230 the partition given in the Torah is optimal. Rather than artificially trying to demonstrate that based on the census data provided by the Torah this in fact was the Levite population, we offer the "possibility" that the population of Levites over the age of 20 years fell within this range. This would happen if approximately 61% of the total male Levite population of 23,000 were over the age of 20 (i.e. 14,030/23,000), and 39% were under. Based on a review of contemporary worldwide population statistics, these proportions are certainly feasible (see for example *The Sex and Age Distribution of the World Populations: The 1992 Revision*, published by The Department of Economic and Social Development). For the period between 1950 and 1990, the percentage of the population under the age of 20 for underdeveloped and industrialized countries ranged from 33% to 54% (Note: the major industrialized countries generally were at the lower end of the range). Thus, while we cannot specifically state the population of each mountain, it is possible that the given partition yielded the smallest cumulative population difference.
23. This apparently violates the agreement they had made with Moshe (*BaMidbar* 32:26-27). See footnote 25.
24. It is, of course, true that Menashe was on Gerizim and, thus, its 52,700 male population (included under Yosef) would also have been diminished. However, the majority of the families from Menashe elected to go to *Erets Yisrael*, and it would require a significantly lopsided manipulation of the contribution of each of the three tribes to the 40,000 man fighting force, and the population of the part of Menashe who elected to enter the land, to make the calculation work.

25. We should mention that the argument presented by R. Yosi ben Bun may not be universally accepted. Firstly, R. Shemuel bar Nachman, quoting R. Yonatan in the same *gemara*, appears to disagree. Secondly, Radak (*Joshua* 22:8) ignores the simple reading of *Joshua* 4:13 and says that based on the agreement cited in *BaMidbar* the entire adult male population of all of the three tribes entered *Erets Yisrael* (Note: He offers no explanation for *Joshua* 4:13.) Finally, *Yalkut Me'am Loez* suggests that all of the men of Gad, Reuven and half of Menashe over the age of 20 initially entered *Erets Yisrael* with the rest of the tribes, but left shortly thereafter (perhaps after participating in the ceremony on Gerizim and Eval) leaving only a contingent of 40,000 men. For the sake of completeness we also note that R. Yosi ben Bun does not suggest how to reconcile the fact that only 40,000 men from the two and one half tribes entered *Erets Yisrael* with their promise to Moshe that only "our children and wives" (*BaMidbar* 32:26) would remain in the *Ever haYarden*.
26. *BaMidbar* 2:1-31.
27. *BaMidbar* 1:19.
28. We must keep in mind that Levi had no land of its own and was given property in the territory of all of the tribes. Menashe had land in *Ever haYarden*, but the greater portion lived in Israel proper.
29. The tribes on the mountain with the blessings were those that formed the greater part of the land of Israel.